Project Report: Design of a Finned Heat Sink for a Luminescent Solar Concentrator

Course: MANE 4730 – Heat Transfer
Project: Semester Design Project

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1. Project Description

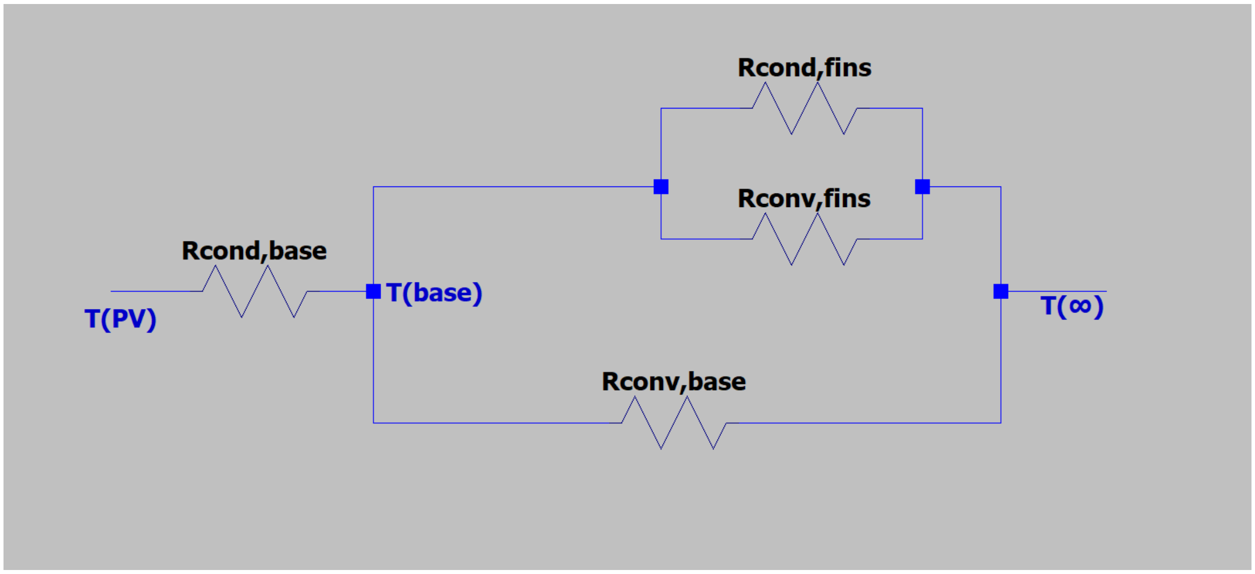
This project focuses on designing a finned heat sink for a Luminescent Solar Concentrator (LSC) system to maintain the photovoltaic (PV) cell temperature at or below 35°C under worst-case conditions. The LSC, integrated into a 1 m × 1 m building facade panel, concentrates sunlight with a factor of 1.2 onto a 1 cm × 1 m PV strip, absorbing 90% of incident light (1000 W/m^2) and converting 25% into electricity, with the rest dissipated as heat. The heat sink must manage this thermal load (8.1 W) via natural convection, as conduction to the LSC backing is ignored. Design constraints include a total heat sink height under 1 cm, operating in an ambient temperature of 27°C. Using aluminum (k = 237 W/m·K), the optimized design features 12 rectangular fins, each 1 mm thick, 9 mm tall, spaced 1 mm apart, attached to a 1 mm thick base. The goal is to ensure thermal stability and efficiency of the PV system through effective heat dissipation, supporting the LSC's application in sustainable building envelopes

2. Assumptions

- Steady Stat
- Constant k for aluminum k = 237 $\frac{W}{mK}$
- Closed System
- 1D Heat Flow
- Adiabatic Fin Tips
- Thin Fin Approximation
- Ambient Temp T = 27°C
- Neglecting Radiation Effects
- Neglecting Wind Effects
- Heat Transfer Mode
- Uniform Heat Generation
- Laminar Flow Nu = 0.59 $Ra^{0.25}$

3. System Thermal Resistance Network

The thermal resistance network maps how heat travels from the PV to the air, starting at the PV temperature and moving through the base's conduction resistance to reach the base temperature. From there, heat splits into two paths: one through the fins, facing convection resistance from the fin array, and another through the unfinned base area, with its own convection resistance, both leading to the ambient air temperature. This setup shows how the base and fins work together to let heat escape, with each resistance acting like a barrier that the heat has to push through to cool the PV down.



4. Analysis

4.1 Symbolic Analysis to Determine the PV Temperature

To determine the PV temperature (T_{PV}) symbolically, we analyze the heat transfer from the PV strip through a finned heat sink to the ambient air under steady-state conditions. The heat generated in the PV (P_{heat}) is dissipated via conduction through the heat sink base and convection from the fins and unfinned base area. The convective heat transfer coefficient (h) is assumed known.

The fin length (L) is how tall each fin grows from the base, giving more area to release heat into the air, but it has to fit within a tight height limit while still cooling the PV enough. Fin thickness (t) decides how wide each fin is, helping heat move through it better if thicker, though it leaves less room for more fins across the PV. The fin width (w) matches the PV's length to spread heat evenly along its full size, making sure the fins and base use the whole PV area to cool it. Fin spacing (s) is the gap between fins, setting how many can fit and how well air flows through too close cramps the air, too far cuts the cooling area. The number of fins (N) controls how much total surface there is to lose heat, limited by the PV's width, balancing more fins for cooling against keeping enough space for air to move. Base thickness (t_{base}) is the layer between the PV and fins, carrying heat to them; a thicker base adds resistance but holds everything together, so it needs to be thin yet strong within the height cap. Lastly, the PV width (W_{PV}), set by the PV strip itself, fixes the heat sink's width, shaping how the heat comes in and how the fins must line up to handle it.

Fin length: L

Fin thickness: t

Fin width: w (equal to PV length)

Fin spacing: s

Number of fins: N

Base thickness: t_{base}

PV width: W_{PV}

The single fin cross-sectional area (A_c) shows the fin's thickness and width combined with its length, helping heat flow from the base to the tip, though with adiabatic tips, we focus on the sides for cooling. while A_b is the space between fins times the width, showing where heat escapes directly from the base to the air, tying into how many fins fit and how much room is left.

Single fin cross-sectional area: $A_c = 2Lw + tw$

Single fin surface area (adiabatic tips): $A_b = s(N-1)$

Unfinned base area: $A_b = s(N-1)w$

The total heat transfer area combines the surface of all the fins with the uncovered base space, showing the whole area that releases heat to the air. The fin parameter blends the heat transfer strength, material properties, and fin thickness to describe how well heat travels through each fin. The heat transfer per fin depends on those factors plus the temperature difference between the base and the air, adjusted by the fin's length and shape, to measure how much heat each fin gets rid of.

Total heat transfer area: $A_t = A_f N + A_b$

Fin parameter: $m = \sqrt{\frac{2h}{kt}}$

Heat transfer per fin: $q_f = \sqrt{2hktw^2}\,(T_b - T_\infty)\tanh(mL)$

Fin efficiency shows how well each fin passes heat to the air compared to a perfect fin, based on its material, thickness, length, and how heat moves through it. Overall surface efficiency combines the efficiency of all the fins with the base area, adjusting for how many fins there are and how much they help, to give a total picture of how the whole heat sink surface works to cool things down.

Fin efficiency: $\eta_f = \frac{\sqrt{2kt}}{2hL}\tanh\left(\sqrt{\frac{2h}{kt}}\,L\right)$

Overall surface efficiency: $\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$

The total heat rate shows all the heat leaving the heat sink, combining what each fin releases with the heat escaping from the bare base, driven by the temperature difference between the base and the air. Base conduction describes how heat moves from the PV to the fins through the base, depending on its thickness, material, and the PV's size, acting like a barrier heat has to cross. Total convection resistance measures how hard it is for heat to leave the whole surface into the air, based on the surface's efficiency and area, while the total thermal resistance adds this to the base's resistance, giving the full challenge heat faces going from the PV to the surroundings.

Total heat rate: $Q_t = Nq_f + hA_b(T_b - T_\infty)$

Base conduction: $R_{cond,base} = \frac{t_{base}}{kW_{PV}w}$

Total convection resistance: $R_{t,o} = \frac{1}{\eta_o h A_t}$

Total thermal resistance: $R_{total} = R_{cond,base} + R_{t,o}$

The PV temperature tells us how hot the photovoltaic strip gets, starting from the surrounding air temperature and adding the extra heat it produces, adjusted by how tough it is for that heat to escape through the heat sink. It wraps up everything on how much heat the PV makes and how well the heat sink, with its base and fins, into one value that shows if the design keeps the PV cool enough to work properly.

PV Temperature: $T_{PV} = T_\infty + P_{heat}R_{total}$

4.2 Determination of Heat Transfer Coefficient on the PV Face Without Fins

For calculation on the heat transfer coefficient on the PV face without fins the first step is to find the Rayleigh number $\mathrm{Ra_L}$. This was done in the equation below where g is earth's gravitational constant, β is the thermal expansion coefficient, $\mathrm{T_s}$ is the surface temperature of the panel, T_∞ is the ambient temperature, L is the characteristic length of the panel, ν is the kinematic viscosity, and α is the thermal diffusivity. For a horizontal plate characteristic length is found using the surface area and perimeter of the plate as found in the equation below

$$L = \frac{A}{P}$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

The thermal expansion coefficient β can be found by taking the reciprocal of $\mathrm{T_{film}}$. To calculate the $\mathrm{T_{film}}$ temperature the average of the $\mathrm{T_s}$ surface temperature and the T_∞ ambient temperature is taken.

$$T_{film} = \frac{(T_s + T_\infty)}{2}$$

$$\beta = \frac{1}{T_{film}}$$

The Nusselt number can then be calculated using the vertical plate equation for laminar flow as found in Fundamentals of Heat and Mass Transfer, (6). This uses the Rayleigh number calculated above which is less than 10^9 and the Prandtl number found in tables for air at the $\mathrm{T_{film}}$ temperature.

$$Nu_L = \left(0.825 + \frac{0.387Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}\right)^2$$

Next rearranging the equation for Nusselt number in terms of heat transfer coefficient, we are able to solve for the $\mathrm{h_{pv}}$ of the system.

$$Nu_L = \frac{h_{PV}L}{k} \quad \rightarrow \quad h_{PV} = \frac{Nu_L k}{L}$$

4.3 Determination of Heat Transfer Coefficient on the PV Face with Fins

To determine the heat transfer coefficient in the PV face with fins it starts with calculating using the same Rayleigh number equation. The difference with vertical plates is the characteristic length is the plate's height.

$$Ra_s = \frac{g\beta(T_s - T_\infty)S^3}{\nu\alpha}$$

The same equation for Nusselt number can be used because of a laminar assumption from the rayleigh number

$$Nu_L = \left(0.825 + \frac{0.387Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}\right)^2$$

This allows us to use the same correlation for heat transfer coefficient that ends in h_{PV}

$$Nu_L = \frac{h_{PV}L}{k} \quad \rightarrow \quad h_{PV} = \frac{Nu_L k}{L}$$

5. Select a Heat Sink Design

5.1 Initial T_{PV} calculation

For our initial T_{PV} the initial design parameters that were used areas follows. The base thickness t = .001m, the length of the fins are .009m the width of the fins is .01m and the number of fins are 80. Our iterations are found in the appendix.

$A_c = 2Lw + tw = 2(.009)(.01) + .001(.01) = .0021$

$A_b = s(N-1)w = (11)(.01)(8.23) = .905$

$A_t = A_f N + A_b = .0021(12) + .905 = .931$

$\eta_f = \frac{\sqrt{2kt}}{2hL}\tanh\left(\sqrt{\frac{2h}{kt}}\,L\right) = \frac{\sqrt{2(237)(.01)}}{2(18)(.1)}\tanh\left(\sqrt{\frac{2(18)}{237(.01)}}\,.1\right) = .224$

$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f) = 1 - \frac{12(.0021)}{.931}(1-.224) = .98$

$R_{t,o} = \frac{1}{\eta_o h A_t} = \frac{1}{(.98)(18)(.931)} = .061$

$R_{cond,base} = \frac{t_{base}}{kW_{PV}w} = \frac{1}{237(.01)(.01)} = 42.2$

$R_{total} = R_{cond,base} + R_{t,o} = 42.2 + .061 = 42.261$

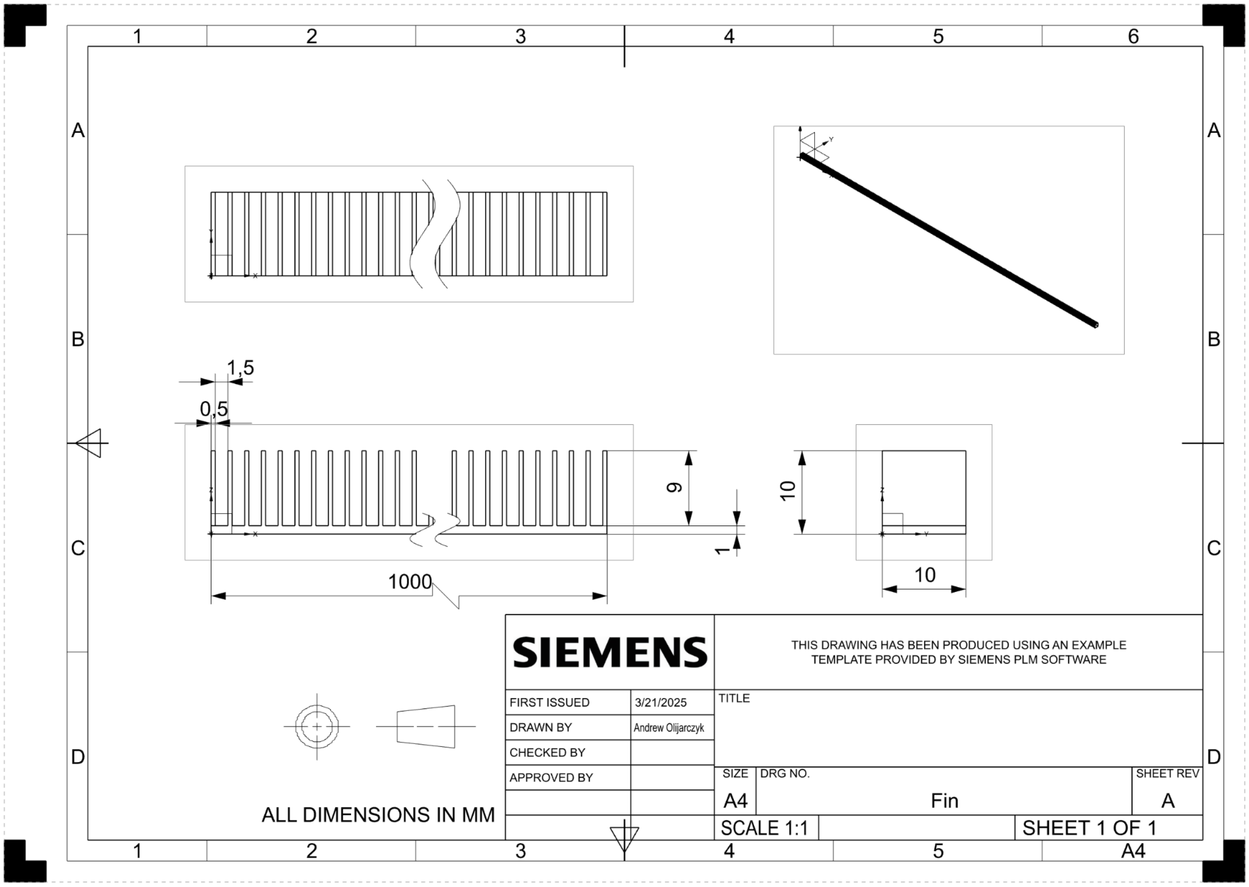
$T_{PV} = T_\infty + R_{total}(W) = 300 + 42.261(8.1) = 369.3$

5.2 Design Iterations to Bring T_{PV} to Specified Value

Final Design Parameters	Magnitude
T_{PV} (in °C) corresponding to this set of parameters	33.53 °C
Material selected	aluminum
Fin Length, L (mm)	9 mm
Fin Width, w (mm)	1 cm
Fin Thickness, t (mm)	.5 mm

To keep the PV temperature below 35°C, we tested different heat sink designs to handle the heat coming from the PV strip while staying within the size limits. We chose aluminum for its good heat-conducting ability, low cost, and ease of making, and started with a basic setup for the fins and base. At first, the PV got too hot because there wasn't enough area to release the heat into the air. So, we made changes, like adding more fins to increase the cooling surface and making each fin thinner to fit them in, while also ensuring air could still flow between them for natural cooling. We also adjusted the fin height and base thickness to stay under the height limit, and improved how well heat moves from the fins to the air by fine-tuning the design. After several tries, we found a setup that brought the PV temperature down to 33.53°C, meeting the goal. The final design uses aluminum, with fins that are 9 mm tall, 1 m wide to match the PV's length, and 0.5 mm thick, creating an effective cooling system for the PV.

5.3 CAD file



6. Conclusion

This heat sink is designed to enhance thermal dissipation from the PV panel to the surrounding environment. By optimizing the design through iterations, it is able to effectively cool the PV panel under an operating temperature of 35°C. The upper surface has a convective heat transfer coefficient of _____ W/m^2K, while the finned lower surface achieves ______ W/m^2K. Aluminum was chosen as a candidate because of comparable effective thermal conductivity of 237 W/mK, as well as its affordability and ease of manufacturing.

During development, challenges prompted design adjustments to address specific issues. With our original calculations we were unable to get an acceptable PV temperature, however through the iterative process the original parameters were able to be modified resulting in an effective thermal dissipation system. Test and trialing each of the variables to see how the interacted with the end result was crucial to getting correct values.

Some of the viable features that had to be played around with were the thickness of the fin (t), number of fins (N), length of fins (L). We found early on that the surface area needed to be maximized to produce more cooling. Thus, the length of fins was put at the max, leaving the other variables to be perfected. After further discovery we were able to find the best amount of fins as well as their thickness to maximize the surface area to properly cool the PV strip at the correct temperature (below 35°C).

Aluminum was used as it was the most cost effective, copper has a better heat transfer coefficient (k), but does little for this situation, thus aluminum would be more practical.

7. References

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8. Technical Contributions

We started the original design and worked through the appendix as a group but Travis was able to do more of the beginning calculations as he had a less busy week. Andrew was able to pick up the end sections as the week went on, as well as having more technical experience with cad.

Team Member	Contributed Section
Student A (Travis)	Sections 1–4.1, appendix
Student B (Andrew)	4.3, 4.3, 5.3, 6, 7, 8

9. Appendix and Excel Sheet

1. Test Equations

$$Ra_s = \frac{g\beta(T_s - T_\infty)S^3}{\nu\alpha} = \frac{9.81\left(\frac{1}{304}\right)(8)\left(\frac{.009(.01)}{2(.009)+2(.01)}\right)}{1.65 * 10^{-5}(2.28 * 10^{-5})} = 1.625 \; * \; 10^6$$

$$Nu_L = \left(0.825 + \frac{0.387 Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}\right)^2 = \left(0.825 + \frac{0.387(1.625 * 10^6)^{1/4}}{\left[1 + (0.492/.705)^{9/16}\right]^{4/9}}\right)^2 = 130.46$$

$$h_{PV} = \frac{Nu_L k}{L} = \frac{130.46(.0263)}{.009} = 381.23$$

Surface area of fin and base

$$A_{fin} = 2Lw + tw = 0.000185 \text{ m}^2$$

$$A_{base} = tw = 0.0075m2$$

Total fin resistance and heat transfer through fin

$$R_{total,fin} = \frac{1}{\eta_{fin} h A_{fin}}$$

$$q_{fin} = \frac{(T_{base} - T_\infty)}{R_{total,fin}} = \eta_{fin} h A_{fin}(T_{base} - T_\infty)$$

Resistance of base through conduction and heat transfer of the base

$$R_{cond,base} = \frac{t}{kA_{base}}$$

$$q_{base} = \frac{(T_{PV} - T_{base})}{R_{cond,base}} = \frac{kA_{base}(T_{PV} - T_{base})}{t}$$

$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{q_f}{hA_f\Theta_B}$$

$$R_{cond,fins} = \frac{R_{cond,fin}}{N}$$

$$R_{cond,fin} = \frac{\Delta x}{kA}$$

Temperature distribution through fins

$$\frac{d^2\theta}{dz^2} - m^2\theta = 0$$

Excess temp where T(z) represents the fin temperature at distance z, its initial condition at 0 represents the base temp.

$$\theta(z) = T(z) - T_\infty$$

$$\theta(0) = T_b - T_\infty$$

$$m^2 = \frac{hP}{k_f A_c}$$

$$h(\theta) = -kA_c \frac{d\theta}{dz}\Big|_L$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-z) + \left(\frac{h}{mk}\right)\sinh m(L-z)}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL}$$

$$N_{uH} = 0.68 + \frac{0.67R_a^{1/4}}{\left[1 + (0.492/P_r)^{9/16}\right]^{4/9}}$$

$$q_f = \sqrt{hPA_c}\,\theta_b\,\frac{\sin mL + \left(\frac{h}{mk}\right)\cosh mL}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL}$$

Heat Transfer of Total System

$$Q_T = Nq_f + hA_b\Delta T$$

2. Sample Calculations

$t = 0.001$ $N = 80$
$L = 0.009$
$w = 0.01$

$A_f = 2Lw + tw = 0.0021$

$\eta_f = \frac{\sqrt{2 \cdot 237 \cdot 0.01}}{2 \cdot 18 \cdot 0.1} \tanh\left(\sqrt{\frac{2 \cdot 18}{237 \cdot 0.01}} \cdot 0.1\right)$

$\eta_f = 0.224$ $A_t = 0.0021 \cdot 12 + 0.00(11)$

$A_t = 0.931$

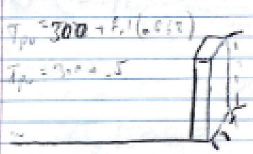
$\eta_o = 1 - \frac{12 \cdot 0.0021}{6.931}(1 - 0.224) = 0.98$

$R_{t,o} = \frac{1}{0.98 \cdot 18 \cdot 0.931} = 0.061$

$R_{cond,base} = \frac{.0025}{237 \cdot 0.01(0.001)} = 42.2$

$T_{pv} = 300 + 8.1(0.063)$

$T_{pv} = 300 + .5$



$T_{pv} = 369.3\,^{\circ}C$

$A_f = 2 \cdot 0.009 \cdot 0.01 + 0.001 \cdot 0.01$

$A_f = 0.00019$ $A_b = 0.0020$

$A_t = 0.00019 \cdot 80 + 0.0092 = 0.0244$

$\eta_f = \frac{\sqrt{2 \cdot 237 \cdot .001}}{2 \cdot 18 \cdot 0.009} \tanh\left(\sqrt{\frac{2 \cdot 18}{237 \cdot .001}} \cdot 0.009\right) = 0.235$

$\eta_o = 1 - \frac{80 \cdot 0.00019}{0.0244}(1 - 0.235) = 0.523$

$R_{cond,base} = \frac{.001}{237 \cdot 0.01 \cdot 0.01} = 0.0422$

$R_{t,o} = 4.35$ $T_{pv} = 300 + 8.1(4.39) = 62.6\,^{\circ}C$

$N = 120$ $t = 0.002$

$A_f = 0.00019$ $A_b = 0.0066$ $A_t = 0.0316$

$\eta_f = \frac{\sqrt{2 \cdot 237 \cdot 0.001}}{2 \cdot 18}$

$A_f = 0.0002$ $A_b = 0.0088$

$A_t = 0.0208$ $\eta_f = \frac{\sqrt{2 \cdot 237 \cdot 0.002}}{2 \cdot 18 \cdot 0.009} \tanh\left(\sqrt{\frac{2 \cdot 18}{237 \cdot 0.002}} \cdot 0.009\right)$

$\eta_f = 0.235$ $\eta_o = 1 - \frac{60 \cdot 0.0002}{0.0208}(1 - 0.235)$

$\eta_o = 0.323$

$A_f = 0.00028$ $A_b = 0.004$

$A_t = 0.0208$ $\eta_f = \frac{\sqrt{2 \cdot 237 \cdot 0.01}}{2 \cdot 18 \cdot 0.009} \tanh\left(\sqrt{\frac{2 \cdot 18}{237 \cdot 0.01}} \cdot 0.009\right)$

Same

$\eta_f = 0.235$

$\eta_o = 1 - \frac{60 \cdot 0.00028}{0.0208}(1 - 0.235)$

$A_f = 0.00028$ $A_b = 0.008$ $A_t = 0.0136$

$\eta_o = 1 - \frac{20 \cdot 0.00028}{0.0136}(1 - 0.235) = 0.685$

$A_f = 0.185$ $A_b = 0.005$

$A_t = 0.01(1 - tN) + N(2 \cdot 0.009 \cdot 0.01) + t \cdot 0.01)$

$A_t = 0.19$

$\eta_f = \frac{\sqrt{2 \cdot 237 \cdot 0.0005}}{2 \cdot 18 \cdot 0.009} \tanh\left(\sqrt{\frac{2 \cdot 18}{237}}\right)$

0.235

$\eta_o = 1 - \frac{1000 \cdot 0.185}{0.19}(1 - 0.235)$

$A_f = 0.0925$ $A_b = 0.0075$

$A_t = 0.1$ $\eta_o = 1 - \frac{500 \cdot 0.0925}{0.0175}(1 - 0.235)$

$\eta_o = .708$

$R_{t,o} = \frac{1}{0.708 \cdot 18 \cdot 0.1} = 0.785$ $R_{cond,base} = \frac{0.0005}{237 \cdot 0.01 \cdot 0.01} = 0.211$

$T_{pv} = 300 + 8.1 \cdot 0.806 = 306.53 = 33.53\,^{\circ}C$