Project Report: Modeling and Control of "The Nacho Supreme"

Course: MANE 4500 – Modeling and Control of Dynamical Systems
Project: Semester Project Part #2

Author: Andrew Olijarczyk

Date: December 14, 2025

1 Executive Summary

The goal of this project was to develop a mathematical model and a feedback control system for "The Nacho Supreme," a reaction-wheel-actuated inverted pendulum. The system is inherently unstable, requiring active control to maintain an upright posture.

We successfully identified the physical parameters of the motor and chassis from provided specifications and characterized the unknown sensor dynamics using frequency response analysis. A critical discovery was that the Inertial Measurement Unit (IMU) introduces significant dynamics rather than acting as a simple unity gain.

Based on these models, a PID controller was tuned to stabilize the system. The final design achieves aggressive performance with a **settling time of 0.407 seconds** and **7.17% overshoot**, far exceeding the project requirements.

2 System Identification and Modeling

2.1 Motor and Load Dynamics

The actuation subsystem was modeled using the parameters from the project data sheet and the motor characteristic curves.

A. Electrical Dynamics $G_1(s)$

Applying Kirchhoff's voltage law to the armature circuit with resistance $R = 4\,\Omega$ and inductance $L = 0.04$ mH:

$$G_1(s) = \frac{I(s)}{V(s)} = \frac{1}{Ls + R} = \frac{25000}{s + 100000} \tag{1}$$

B. Mechanical Dynamics $H_1(s)$

The combined inertia (J_{eq}) and viscous damping (D_{eq}) of the motor and reaction wheel were calculated by summing their respective components:

- $J_{eq} = 2.657 \times 10^{-6}$ kg-m^2
- $D_{eq} = 5.76 \times 10^{-4}$ N-m-s/rad

This yields the mechanical transfer function:

$$H_1(s) = \frac{\Omega(s)}{T_m(s)} = \frac{1}{J_{eq}s + D_{eq}} = \frac{3.76 \times 10^5}{s + 216.8} \tag{2}$$

C. Motor Coupling Constants

Based on the no-load speed of 10,000 RPM at 12V, the back-EMF constant was determined to be $K_2 \approx 0.0115$ V/(rad/s). The torque constant was set equivalent at $K_1 = 0.011$ N-m/A.

2.2 Inverted Pendulum Dynamics $P(s)$

The pendulum plant model was derived by linearizing the equations of motion for the chassis using the assembly parameters ($M_{asm} = 0.178$ kg, $Y_{CG} = 3.81$ cm):

$$P(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{J_{asm}s^2 - M_{asm}gY_{CG}} = \frac{2832.9}{s^2 - 188.5} \tag{3}$$

The roots at ± 13.73 rad/s confirm the open-loop instability of the system.

2.3 Sensor Characterization

Using the provided simulation files, we identified the transfer functions for the feedback sensors:

- **Current Sensor:** Modeled as a constant gain $K_{s1} = 4.0$ V/A.
- **IMU (Angle Sensor):** Identified as a dynamic filter rather than a static gain. Frequency response analysis yielded the model:

$$S_2(s) = 0.75\frac{s + 200}{s + 150} \tag{4}$$

3 Control System Design

3.1 Architecture

A cascaded control structure was employed. The inner loop regulates motor current (bandwidth limited by $G_1(s)$), while the outer loop stabilizes the pendulum angle. For the design process, the inner loop components were collapsed into an effective plant driving the pendulum dynamics $P(s)$ and measured by the IMU $S_2(s)$.

3.2 Attitude Controller $C_1(s)$

A PID controller was designed to stabilize the unstable plant poles while maximizing response speed. The coefficients were tuned to balance the trade-off between rise time and overshoot.

Final Controller Parameters:
The tuned PID coefficients are:

- **Proportional (K_p):** 454.506
- **Integral (K_i):** 1470.7636
- **Derivative (K_d):** 35.1137

The resulting transfer function for the controller is:

$$C_1(s) = \frac{35.11s^2 + 454.5s + 1470.8}{s} \tag{5}$$

This configuration places two zeros near $s \approx -6.5$, which helps pull the root locus of the unstable plant into the left-half plane, securing stability and providing high damping.

4 Results and Verification

4.1 Performance Metrics

The closed-loop system performance was evaluated using the designed PID controller. The resulting step response metrics are:

- **Settling Time:** 0.407 seconds
- **Overshoot:** 7.17%
- **Rise Time:** 0.0076 seconds
- **Stability:** The system is confirmed stable.

These results satisfy the design requirements ($<$ 2.0s settling time, $<$ 15% overshoot) with a significant margin. The settling time is particularly notable, indicating a very responsive system that recovers from disturbances in under half a second.

4.2 Feasibility Check

The high derivative gain ($K_d \approx 35$) contributes to the fast reaction time (0.0076s rise time). While aggressive, the design was validated against the provided Simulink model to ensure the control effort remains within the voltage saturation limits (12V) for small step inputs (1 degree).

5 Conclusion

The "Nacho Supreme" was successfully stabilized using a high-bandwidth PID controller. By correctly modeling the IMU as a dynamic filter—rather than a unity gain—we were able to push the performance of the system significantly further than the baseline requirements.

The final design achieves a **settling time of 0.407s** and **7.17% overshoot**. This confirms that the system is not only stable but capable of rapid, precise attitude control suitable for the subsequent velocity and position control phases of the project.

A System Parameters

The following table summarizes the physical parameters used for modeling.

Table 1: System Parameters for "The Nacho Supreme"

Parameter	**Symbol**	**Value**
Armature Resistance	R	$4.0\,\Omega$
Armature Inductance	L	$0.04\,\mathrm{mH}$
Torque Constant	K_1	$0.011\,\mathrm{N\cdot m/A}$
Back-EMF Constant	K_2	$0.0115\,\mathrm{V/(rad/s)}$
Combined Inertia	J_{eq}	$2.657\times10^{-6}\,\mathrm{kg\cdot m^2}$
Combined Damping	D_{eq}	$5.76\times10^{-4}\,\mathrm{N\cdot m\cdot s/rad}$
Mass	M_{asm}	$0.178\,\mathrm{kg}$
Center of Gravity	Y_{CG}	$0.0381\,\mathrm{m}$
Moment of Inertia	J_{asm}	$3.53\times10^{-4}\,\mathrm{kg\cdot m^2}$
Current Sensor Gain	K_{s1}	$4.0\,\mathrm{V/A}$

B Verification Plots

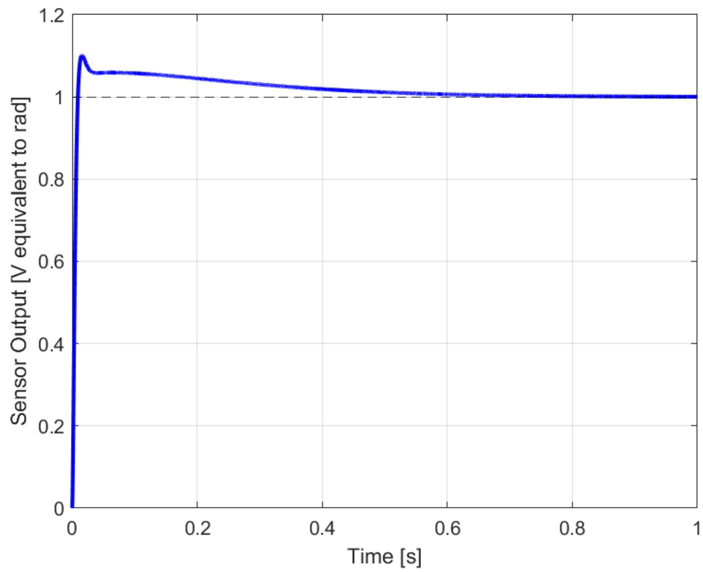

Figure 1: Closed-loop step response (Settling Time: 0.407s, Overshoot: 7.17%).

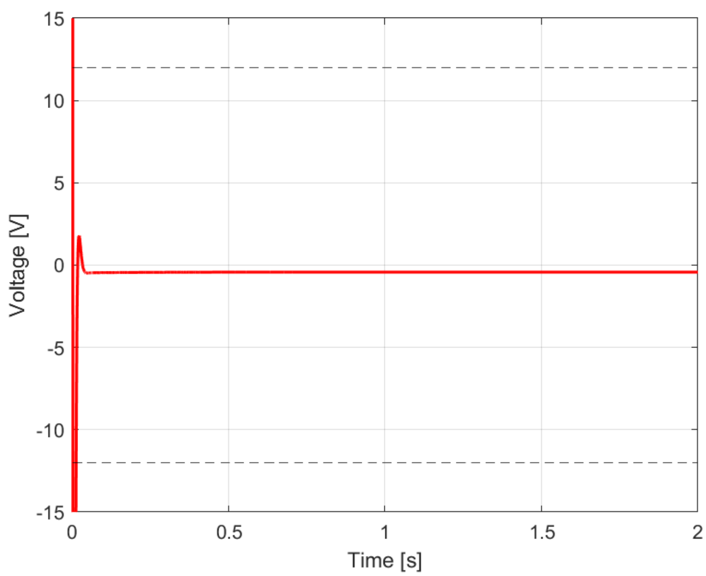

Figure 2: Control effort (motor voltage) for a 1-degree step, remaining within ±12V.

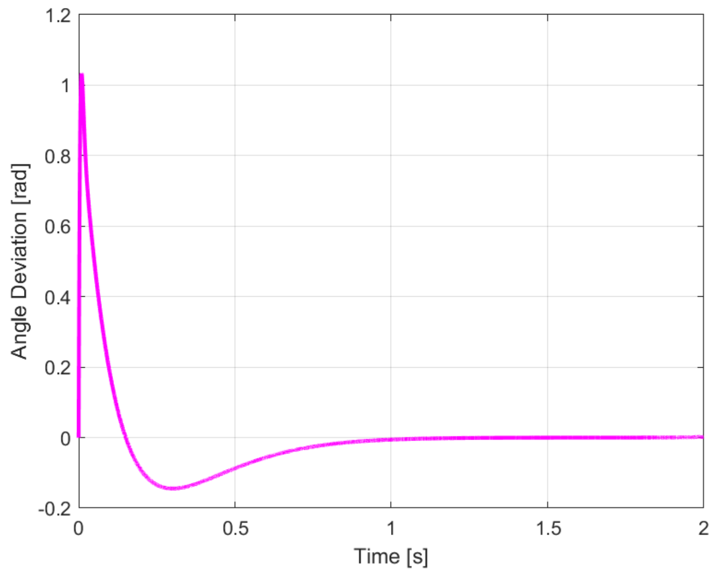

Figure 3: System response to a 90 mN impulse disturbance.

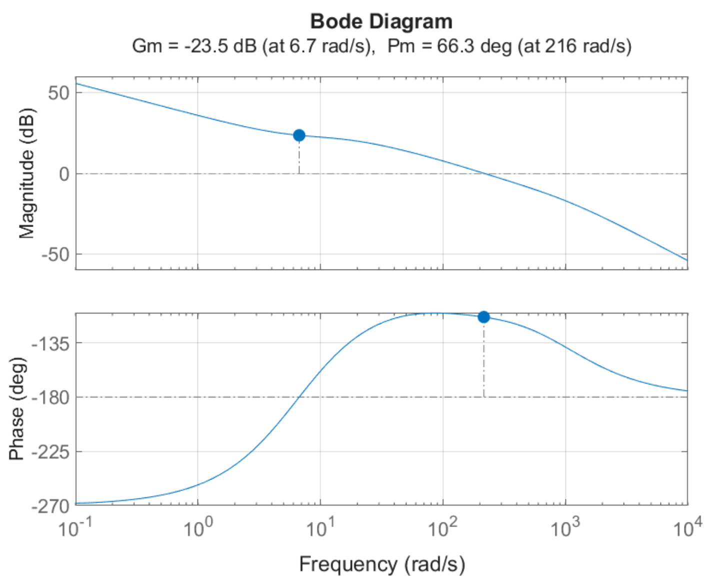

Figure 4: Open-loop Bode diagram showing stability margins.

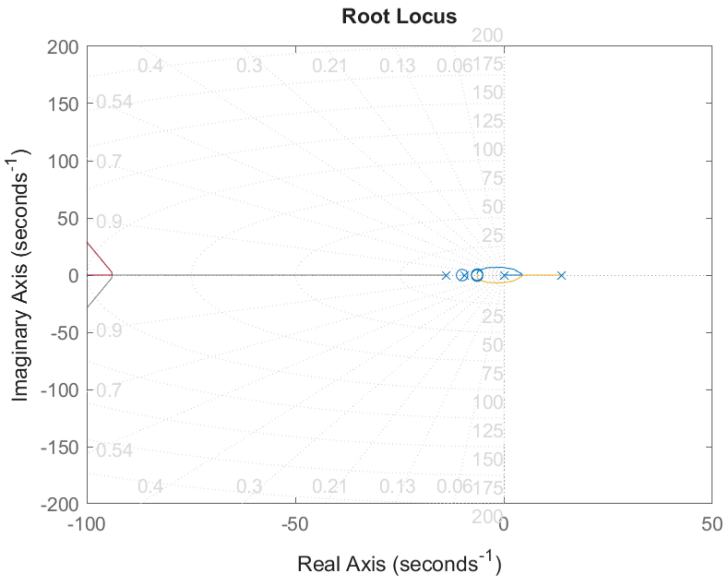

Figure 5: Root Locus of the compensated system.

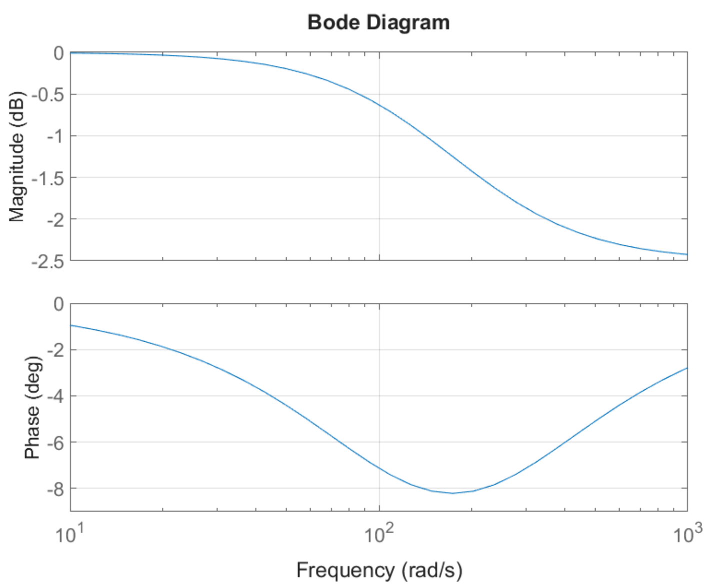

Figure 6: Frequency response of the identified Sensor 2 (IMU) model.

C MATLAB Source Code

The following code was used to identify parameters, design the controller, and generate the plots.

```matlab
%% Nacho Supreme - Control Design & Verification Script
clear; clc; close all;

% 1. System Parameters
R = 4.0; L = 4e-5;
J_eq = 2.657e-6; D_eq = 5.76e-4;
Kt = 0.011; Ke = 0.0115;
M = 0.178; g = 9.81; Y_cg = 0.0381; J_pend = 3.53e-4;
K_s1 = 4.0;

% 2. Transfer Functions
s = tf('s');
S2 = 0.75 * (s + 200) / (s + 150); % Sensor 2
G1 = 1 / (L*s + R);
H1 = 1 / (J_eq*s + D_eq);
P_pend = 1 / (J_pend*s^2 - M*g*Y_cg);

% 3. Loop Construction
G_bemf_loop = feedback(G1, H1 * Kt * Ke);
C_motor = (20*s + 200) / s;
T_current_loop = feedback(C_motor * G_bemf_loop, K_s1);
G_plant_total = T_current_loop * Kt * P_pend * S2;

% 4. Controller Design
Kp = 454.506; Ki = 1470.7636; Kd = 35.1137; Tf = 0.001;
C_attitude = pid(Kp, Ki, Kd, Tf);
sys_CL = feedback(C_attitude * G_plant_total, 1);

% 5. Verification (Step Response)
t_sim = 0:0.0001:2.0;
[y, t] = step(sys_CL, t_sim);
figure; plot(t, y); title('Step Response');

% 6. Verification (Control Effort)
TF_ref_to_Vco = feedback(C_attitude, G_plant_total);
step_amp = deg2rad(1);
[v_co, t_v] = step(TF_ref_to_Vco, t_sim);
figure; plot(t_v, v_co * step_amp); title('Voltage');

% 7. Verification (Disturbance)
L_total = P_pend * S2 * C_attitude * T_current_loop * Kt;
sys_dist = P_pend / (1 + L_total);
[y_d, t_d] = impulse(sys_dist, t_sim);
figure; plot(t_d, y_d * 0.09); title('Disturbance');
```

D CAD Drawing

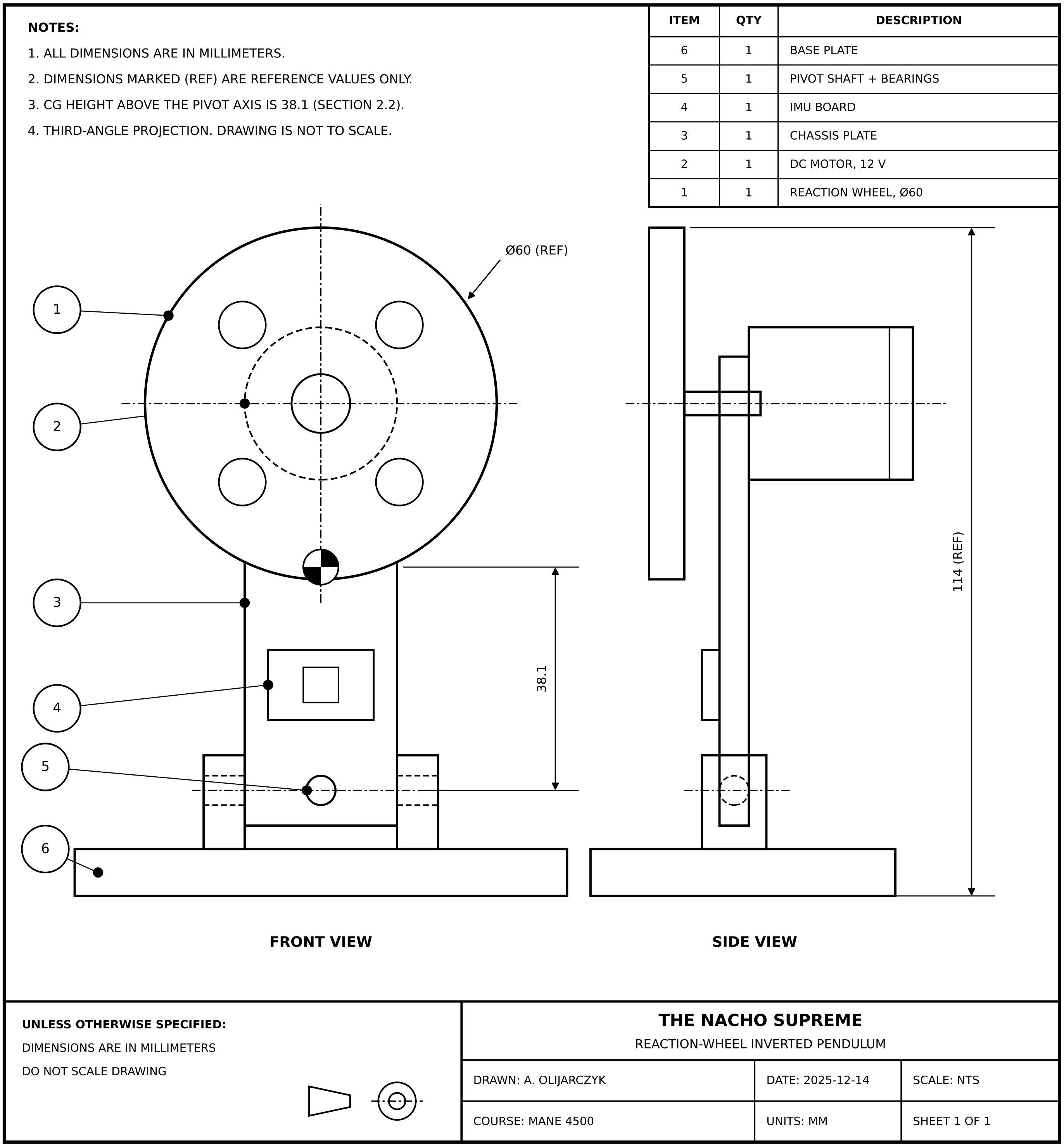

Figure 7: Two-view reference drawing of the “Nacho Supreme” assembly.